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LETTER TO THE EDITOR

Perturbed self-dual Chern–Simons vortices

Dileep P Jatkar and Avinash Khare
Institute of Physics, Sachivalaya Marg, Bhubaneswar-751005, India

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Abstract. We consider small perturbations around the self-dual Chern–Simons as well as Maxwell Chern–Simons vortices. For the topological case, we show that there exist perturbations for which, at least to leading order, the vortices remain non-interacting. On the other hand, for the non-topological case, we show that for most of the perturbations the vortices are unstable against decay to the elementary excitations.

In last few years the charged vortex solutions [1] of the Abelian Higgs model with Chern–Simons (CS) term [2] have received considerable attention in the literature. Sometime ago we showed that these charged vortex solutions continue to exist even in the absence of the gauge field kinetic energy term [3]. Later a significant advance was made by Hong *et al* [4] as well as by Jackiw and Weinberg [5] who showed that self-dual CS vortices can be obtained in the case where the Higgs potential is

$$V(\phi)=\frac{e^4}{8\mu^2}|\phi^2|(|\phi|^2-v^2)^2 \tag{1}$$

which corresponds to being at the first-order transition point. It has also been shown that these vortices are non-interacting and that there is a $N=2$ supersymmetry in the problem [6]. Further in this case one also has self-dual non-topological vortices [7, 8]. These non-topological vortices are at the threshold of their stability against decay to the elementary excitations. Sum rules have recently been derived by one of us for both types of self-dual vortices and using them the magnetic moment has been computed exactly [8, 9]. Finally, Lee *et al* [10] have shown that self-dual vortices can also be obtained in Maxwell CS theory if one couples an additional neutral scalar field to the Higgs field.

The purpose of this letter is to consider small perturbations around self-dual CS as well as Maxwell CS vortices. In particular we consider small perturbations in the Higgs potential. In the topological case we show that there exists a perturbation for which, at least to leading order, the vortices remain non-interacting. On the other hand, for the non-topological case we show that for most of the perturbations the vortices are unstable against decay to the elementary excitations.

We shall first consider perturbations around the self-dual CS vortices. One starts with

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu-\mathrm{i}eA_\mu)\phi^*(\partial^\mu+\mathrm{i}eA^\mu)\phi+\frac{\mu}{4}\varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda-V(\phi) \tag{2}$$

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where $V(\phi)$ is given in (1). On using the ansatz ($\rho \geqslant 0, 0 \leqslant \theta \leqslant 2\pi$)

$$A(\boldsymbol{\rho}) = -e_\theta \frac{ev^2}{r\mu}(g(r) - n) \qquad A_0(\boldsymbol{\rho}) = \frac{ev^2}{\mu} h(r)$$
$$\phi(\boldsymbol{\rho}) = \exp(in\theta) v f(r) \qquad \rho = \frac{\mu r}{e^2 v^2} \tag{3}$$

where r, g, h and f are dimensionless variables, the energy of the n-vortex can be shown to be

$$E_n = \pi v^2 \int_0^\infty r\,\mathrm{d}r \left[\left(\frac{\mathrm{d}f}{\mathrm{d}r} - \frac{gf}{r}\right)^2 + \left(\frac{1}{rf}\frac{\mathrm{d}g}{\mathrm{d}r} - \frac{f}{2}(f^2-1)\right)^2 + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(g(f^2-1))\right] \tag{4}$$

where use has been made of Gauss's law

$$-hf^2 = \frac{1}{r}\frac{\mathrm{d}g}{\mathrm{d}r}. \tag{5}$$

The self-dual equations (for $n \geqslant 0$) which have minimum energy emerge from here. They are

$$\frac{\mathrm{d}f}{\mathrm{d}r} = \frac{gf}{r} \tag{6a}$$

$$\frac{1}{r}\frac{\mathrm{d}g}{\mathrm{d}r} = \frac{f^2}{2}(f^2-1). \tag{6b}$$

The self-dual equations for $n < 0$ can be trivially obtained by letting $g \to -g$, $f \to f$ and $h \to -h$. We shall discuss topological and non-topological cases separately.

(a) Topological case. In this case energy of the self-dual n-vortex is

$$E_n = \pi v^2 n \tag{7}$$

from where it follows that the vortices are non-interacting ($E_n = nE_1$). Their flux Φ, charge Q, angular momentum J (which is in general fractional) and magnetic moment K_z is given by

$$\Phi = \frac{2\pi}{e} n \qquad Q = \mu\Phi \qquad J = -\frac{\pi\mu}{e^2} n^2 \qquad K_z = \frac{2\pi\mu^2}{e^3 v^2}(n^2 + n). \tag{8}$$

The magnetic moment has been calculated by making use of the following two sum rules [8]

$$n = \frac{1}{2}\int_0^\infty f^2(1-f^2) r\,\mathrm{d}r \tag{9}$$

$$n^2 = \frac{1}{2}\int_0^\infty (1-f^2)^2 r\,\mathrm{d}r. \tag{10}$$

Let us now study the effect of perturbing the topological self-dual vortex by

$$-\mathscr{L}_{\text{pert}} = V_{\text{pert}} = \alpha \frac{e^4 v^2}{\mu^2} |\phi|^2 (v^2 - |\phi|^2) \tag{11}$$

where $\alpha \ll 1$. On using (1) and (11) it easily follows that, irrespective of the sign of α, the system now corresponds to being below the first-order transition point. In general if

$$V(f) = Bf^2 - |A| f^4 + Cf^6 \tag{12}$$

then it can be shown that $4BC \leqslant (>) A^2$ corresponds to $T \leqslant (>) T_c^1$ [11]. Thus $f = 0$ is now a local minimum while the two absolute degenerate minima to $O(\alpha)$ are at

$$f = \pm(1 + 2\alpha). \tag{13}$$

However, due to the topological nature of the vortex solution, $g(0)$, $g(\infty)$ and $f(0)$ must remain unaltered to all orders in α, i.e.

$$g(0) = n \qquad g(\infty) = 0 \qquad f(0) = 0. \tag{14}$$

The profiles of g, h and f will, however, get modified from their self-dual configurations. On substituting

$$\begin{aligned} f(r) &= f_{\mathrm{sd}}(r) + \alpha f_1(r) + O(\alpha^2) \\ g(r) &= g_{\mathrm{sd}}(r) + \alpha g_1(r) + O(\alpha^2) \\ h(r) &= h_{\mathrm{sd}}(r) + \alpha h_1(r) + O(\alpha^2) \end{aligned} \tag{15}$$

in the energy expression (4) we find that the self-dual n-vortex energy is unaltered to order α. This is because in view of (6) the first two terms in (4) do not contribute to order α while the last term in (4) always contribute $\pi v^2 n$. Hence, the energy of the perturbed n-vortex to $O(\alpha)$ is given by

$$E_n = \pi v^2 n + 2\alpha\pi v^2 \int_0^\infty r\,\mathrm{d}r\, f_{\mathrm{sd}}^2 (1 - f_{\mathrm{sd}}^2). \tag{16}$$

On using the sum rule (9) we find then that

$$E_n = \pi v^2 n (1 + 4\alpha). \tag{17}$$

Thus due to the perturbation, the n-vortex energy increases or decreases depending on the sign of α. However, in both the cases one finds that to $O(\alpha)$ the vortices are still non-interacting ($E_n = nE_1$), even though in either case one is now away from the self-dual point.

Let us consider another perturbation given by

$$-\mathscr{L}_{\mathrm{pert}} = V_{\mathrm{pert}} = \alpha \frac{e^4 v^2}{\mu^2} (v^2 - |\phi|^2)^2 \tag{18}$$

which has recently been discussed by Bezeia [12]. As has been shown by him, if $\alpha > 0$ then the situation corresponds to being below the first-order transition point (and hence one has topological vortices). Using the sum rule (10) one then recovers his result

$$E_n = \pi v^2 n (1 + 4n\alpha). \tag{19}$$

Thus in this case

$$E_n - nE_1 = 4\pi v^2 \alpha n (n-1) > 0 \tag{20}$$

so that the vortex–vortex interaction is now repulsive in nature. Not surprisingly one finds that in this case the Ginzburg–Landau parameter

$$K_{\mathrm{GL}}=\frac{m_s}{m_v}=\frac{(1+28\alpha)e^2v^2/\mu}{(1+16\alpha)e^2v^2/\mu}>1 \tag{21}$$

so that one is in the type-II region.

Clearly one can consider several other perturbations of the form $\alpha(1-f^2)(a+bf^2)$ for which energy can be computed exactly to $\mathrm{O}(\alpha)$ with the help of the sum rules (9) and (10).

(b) Non-topological case. In this case the energy of the self-dual non-topological n-vortex is

$$E_n=\pi v^2(n+\beta) \tag{22}$$

where $g(\infty)=-\beta$ with β being any positive number satisfying $\beta>n+2$ [9]. This has been proved by one of us using the following sum rules [9]

$$n+\beta=\frac{1}{2}\int_0^\infty f^2(1-f^2)r\,\mathrm{d}r \tag{23}$$

$$\beta^2-n^2=\frac{1}{2}\int_0^\infty (2f^2-f^4)r\,\mathrm{d}r. \tag{24}$$

The flux Φ, charge Q, angular momentum J and magnetic moment K_z of these non-topological vortices are given by

$$\begin{gathered}\Phi=\frac{2\pi}{e}(n+\beta)\qquad Q=\mu\Phi\qquad J=\frac{\pi\mu}{e^2}(\beta^2-n^2)\\ K_z=-\frac{2\pi\mu^2}{e^3v^2}(\beta+n)(\beta-n-1).\end{gathered} \tag{25}$$

Further, the mass of the elementary excitation in the theory is given by

$$m_s=\frac{e^2v^2}{2\mu} \tag{26}$$

so that at the self-dual point the non-topological vortices are at the threshold of their stability against decay to the elementary excitations, i.e.

$$\frac{E_{\mathrm{sd}}}{Q_{\mathrm{sd}}}=\frac{m_s}{e}=\frac{ev^2}{2\mu}. \tag{27}$$

Let us now study the effect of perturbing the non-topological self-dual vortex by

$$-\mathscr{L}_{\mathrm{pert}}=V_{\mathrm{pert}}=\frac{\alpha e^4v^4}{\mu^2}|\phi|^2 \tag{28}$$

where $0<\alpha\ll 1$. Using (1), (12) and (28) it follows that for $\alpha>0$ this situation corresponds to $T>T_c^1$, i.e. $f=0$ is now the absolute minimum of the theory so that, to all orders in $\alpha, f(\infty)=0$. Similarly it is clear that to all orders in α, $g(0)=n$. However, $g(\infty)$ may get altered from its unperturbed value of $-\beta$. Hence, the self-dual non-topological vortex energy may get modified to $\mathrm{O}(\alpha)$. For the same reason Φ, Q, J and K_z may also get modified. However, it is important to notice that the ratio $E_{\mathrm{sd}}/Q_{\mathrm{sd}}$ is unaltered to $O(\alpha)$. Hence, the energy per unit charge of the perturbed non-topological vortex to $O(\alpha)$ is given by

$$\frac{E}{Q}=\frac{ev^2}{2\mu}+\frac{\alpha ev^2}{(n+\beta)\mu}\int_\mu^\infty r\,\mathrm{d}r f_{\mathrm{sd}}^2. \tag{29}$$

On using sum rules (23) and (24) this simplifies to

$$\frac{E}{Q}=\frac{ev^2}{2\mu}[1+4\alpha(\beta-n-1)]. \tag{30}$$

Because of the perturbation (28) the scalar meson mass m_s is also increased and is given by

$$m_\mathrm{s}=(1+4\alpha)\frac{ev^2}{2\mu}. \tag{31}$$

However, since $\beta>n+2$ [9] it immediately follows that $E/Q>m_\mathrm{s}/e$ so that with this perturbation the non-topological vortex is unstable against decay into the elementary excitations.

Let us consider another perturbation given by

$$-\mathscr{L}_{\mathrm{pert}}=V_{\mathrm{pert}}=\frac{\alpha e^4 v^4}{\mu^2}|\phi|^4 \tag{32}$$

where $0<\alpha\ll 1$. This situation again corresponds to $T>T_\mathrm{c}^\mathrm{l}$. Following the arguments of the previous perturbation it follows that to $O(\alpha)$, the energy per unit charge of the perturbed non-topological vortex is given by

$$\begin{aligned}\frac{E}{Q}&=\frac{ev^2}{2\mu}+\frac{\alpha ev^2}{(n+\beta)\mu}\int_0^\infty r\,\mathrm{d}r f_{\mathrm{sd}}^4\\&=\frac{ev^2}{2\mu}[1+4\alpha(\beta-n-2)].\end{aligned} \tag{33}$$

Since the $|\phi|^4$ perturbation does not change the mass of the scalar field, m_s/e continues to have the value $ev^2/2\mu$. Therefore even with this perturbation the non-topological vortex is unstable against decay into the elementary excitations. It also follows that for any perturbation of the form $\alpha f^4+\gamma f^6$ ($\alpha>0$, $\gamma>0$), the vortex will be unstable.

Finally we consider the perturbation

$$-\mathscr{L}_{\mathrm{pert}}=V_{\mathrm{pert}}=-\frac{\alpha e^4}{8\mu^2}|\phi|^2(|\phi|^2-v^2)^2 \tag{34}$$

($0<\alpha\ll 1$). Unlike the previous perturbations this situation still corresponds to $T=T_\mathrm{c}^\mathrm{l}$. In this case using (4) one can show that

$$\frac{E}{Q}>\frac{ev^2}{2\mu}\left(1-\frac{\alpha}{2}\right) \tag{35}$$

while

$$\frac{m_\mathrm{s}}{e}=\frac{ev^2}{2\mu}\left(1-\frac{\alpha}{2}\right) \tag{36}$$

so that even in this case the non-topological vortex is unstable.

Finally let us consider perturbations around the self-dual Maxwell CS vortices. We start with

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}(\partial_\mu-\mathrm{i}eA_\mu)\phi^*(\partial^\mu+\mathrm{i}eA^\mu)\phi+\tfrac{1}{8}\partial_\mu N\partial^\mu N$$
$$+\frac{\mu}{4}\varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda-\tfrac{1}{8}e^2N^2|\phi|^2-\tfrac{1}{8}(e|\phi|^2-ev^2+\mu N)^2 \tag{37}$$

where N is a neutral scalar field. On using the ansatz

$$\begin{aligned} \boldsymbol{A}(\boldsymbol{\rho})&=-\boldsymbol{e}_\theta\frac{A(\rho)}{\rho} \qquad & A_0(\boldsymbol{\rho})&=A_0(\rho)\\ \phi(\boldsymbol{\rho})&=\exp(\mathrm{i}n\theta)f(\rho) \qquad & N(\boldsymbol{\rho})&=N(\rho)\end{aligned} \tag{38}$$

the energy of the n-vortex can be shown to be

$$E_n=\pi\int_0^\infty\rho\,\mathrm{d}\rho\left[\left(\frac{\mathrm{d}A_0}{\mathrm{d}\rho}-\frac{1}{2}\frac{\mathrm{d}N}{\mathrm{d}\rho}\right)^2+\left(\frac{1}{\rho}\frac{\mathrm{d}A}{\mathrm{d}\rho}-\frac{1}{2}(ef^2-ev^2+\mu N)\right)^2\right]$$
$$+\pi\int_0^\infty\rho\,\mathrm{d}\rho\left[\left(\frac{\mathrm{d}f}{\mathrm{d}\rho}-(n+eA)\frac{f}{\rho}\right)^2+e^2f^2\left(A_0-\frac{N}{2}\right)^2\right.$$
$$\left.+\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}[(n+eA)(f^2-v^2)]\right]. \tag{39}$$

The self-dual equations which follow from here are

$$A_0=\frac{N}{2}\qquad\frac{\mathrm{d}f}{\mathrm{d}\rho}=(n+eA)\frac{f}{\rho}\qquad\frac{1}{\rho}\frac{\mathrm{d}A}{\mathrm{d}\rho}=\frac{1}{2}(ef^2+ev^2+\mu N). \tag{40}$$

Further one has the Gauss law equation

$$\frac{\mathrm{d}^2A_0}{\mathrm{d}\rho^2}+\frac{1}{\rho}\frac{\mathrm{d}A_0}{\mathrm{d}\rho}-e^2A_0f^2=\frac{\mu}{\rho}\frac{\mathrm{d}A}{\mathrm{d}\rho}. \tag{41}$$

From here we obtain the following two sum rules

$$n+\beta=\frac{e^2}{2}\int_0^\infty\rho\,\mathrm{d}\rho(v^2-f^2)-\frac{\mu e}{2}\int_0^\infty\rho\,\mathrm{d}\rho\,N \tag{42}$$

$$n+\beta=\frac{e^3}{2\mu}\int_0^\infty\rho\,\mathrm{d}\rho\,Nf^2 \tag{43}$$

where $\beta=0$ for topological vortices.

Using these two sum rules and the discussion about the CS vortices one can consider the effects of various perturbations. In particular if

$$-\mathscr{L}_{\mathrm{pert}}=V_{\mathrm{pert}}=\alpha ev^2(e|\phi|^2-ev^2+\mu N) \tag{44a}$$

$$-\mathscr{L}_{\mathrm{pert}}=V_{\mathrm{pert}}=\alpha ev^2N|\phi|^2 \tag{44b}$$

then one can show that in either case irrespective of the sign of α the vortices are still topological and non-interacting.

If on the other hand one considers the perturbation

$$-\mathscr{L}_{\mathrm{pert}}=V_{\mathrm{pert}}=\alpha e^2|\phi|^4 \tag{45}$$

then one can show that for $\alpha > 0$ the theory has absolute minima at $\phi = 0$, $N = ev^2/\mu$. Following thc arguments of the CS vortices one can show that in this case the non-topological vortices are unstable against decay to the charged scalars. This is because at the self-dual point $E_{sd}/Q_{sd} = m_s/e$ while with this perturbation $E/Q > m_s/e$. Similarly the perturbation

$$-\mathscr{L}_{\text{pert}} = V_{\text{pert}} = -\alpha e^2 N^2 |\phi|^2 - \alpha(e|\phi|^2 - ev^2 + \mu N)^2 \tag{46}$$

also leads to non-topological vortices for which $E/Q > m_s/e$.

More results could be derived here if one could obtain the analogue of the n^2 sum rule as in the CS vortex case. Even there more results could be obtained if one could derive any other sum rule. In particular one may then be able to improve the bound on β.

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